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University of Kansas, Fall 2019
Philosophy 666: Rational Choice Theory
October 4, 2019

test 1 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Questions:

1. Suppose there are three items – *a*, *b*, and *c* – and a person has the preferences *a* P *b* and *a* P *c*. Suppose the person says, "My preferences satisfy the completeness condition because every item is mentioned in at least one of my preferences." Does this remark reflect a correct understanding of the completeness condition? Why or why not?

   answer:

   No, because the completeness condition requires that a person have a preference (including, possibly, an opinion of indifference) between every pair of items – not just that every item be mentioned in at least one of the person's preferences. (As it happens, the person's preferences do not satisfy the completeness condition, because the person has no preference between *b* and *c*.)

2. Suppose Dwight has the following six preferences:

   s P *k* *s* P *m* *s* P *p* *k* P *m* *m* P *p* *p* P *k*

   Also, suppose Dwight possess item *p*, but none of the others. If Arthur wants to use Dwight as a money pump, which of the following offers should Arthur make to Dwight?

   (a) "I see you have item *p*. If you give me that and 5 cents, I will give you item *k*."

   (b) "I see you have item *p*. If you give me that and 5 cents, I will give you item *s*."

   (c) "I see you have item *p*. If you give me that and 5 cents, I will give you item *m*."

   (d) none of the above – There is no reason to think Dwight can be used as a money pump.

   answer: c

3. Suppose Susan has these preferences: *a* P *b*, *b* P *c*, *c* P *d*. And suppose Susan's preference for *a* over *b* is one third as strong as her preference for *b* over *c*, which in turn is one fifth as strong as her preference for *c* over *d*. What is an interval utility function that accurately represents Susan's preferences?

answer: the following, or any positive linear transformation of it:

x	$u(x)$
a	30
b	29
c	26
d	11

4. Suppose that, tomorrow afternoon, Margaret can either do her laundry or do her taxes. If she does her laundry, then the outcome will be good if the laundromat is not crowded and bad if the laundromat is crowded. If she does her taxes, then the outcome will be great if she has no problems and terrible if she has problems. If you were to set up a standard choice matrix for this situation, which of the following would be the heading for one of the columns in your matrix?

   (a) 'laundromat not crowded'

   (b) 'laundromat not crowded and no problems with taxes'

   (c) 'do laundry'

   (d) 'do laundry and problems with taxes'

   answer: b

5. Suppose Wayne is applying the maximin rule in a particular situation, for which he has written out a choice matrix. Do the numbers in the matrix need to be utilities from an interval utility function, or is it sufficient if they are from an ordinal utility function? Explain your answer.

   answer:

   It is sufficient if they are from an ordinal utility function. The maximin rule only refers to the order in which the agent ranks the various possible outcomes, not the relative strengths of the agent's preferences. Since the former information would be represented in an ordinal utility function, such a function would be sufficient.

The following choice matrix is for questions 6–8.

	S_1	S_2	S_3
A_1	8	5	4
A_2	6	1	7
A_3	2	3	9

6. Which option(s) would be recommended by the optimism-pessimism rule, with an optimism index of 3/4? Show your calculations of the α-indexes of the options. (You do not have to reduce any fractions.)

   answer:

   $$\text{α-index for } A_1 = \left(\frac{3}{4}\right)(max) + \left(1 - \frac{3}{4}\right)(min) = \left(\frac{3}{4}\right)(8) + \left(\frac{1}{4}\right)(4) = \frac{24}{4} + \frac{4}{4} = \frac{28}{4}$$

   $$\text{α-index for } A_2 = \left(\frac{3}{4}\right)(max) + \left(1 - \frac{3}{4}\right)(min) = \left(\frac{3}{4}\right)(7) + \left(\frac{1}{4}\right)(1) = \frac{21}{4} + \frac{1}{4} = \frac{22}{4}$$

   $$\text{α-index for } A_3 = \left(\frac{3}{4}\right)(max) + \left(1 - \frac{3}{4}\right)(min) = \left(\frac{3}{4}\right)(9) + \left(\frac{1}{4}\right)(2) = \frac{27}{4} + \frac{2}{4} = \frac{29}{4}$$

   A_3 has the highest α-index, so it is the option that the optimism-pessimism rule would recommend.

7. Which option(s) would be recommended by the minimax regret rule? Show the regret matrix and how you can use it to identify the option(s) that would be recommended by the minimax regret rule.

   answer:

   regret matrix (with an additional column):

	S_1	S_2	S_3	maximum regret for each option:
A_1	0	0	5	5
A_2	2	4	2	(4)
A_3	6	2	0	6

   The smallest maximum regret is circled. Because it is in the row for A_2, that is the option that the rule would recommend.

8. Which option(s) would be recommended by the rule of maximizing expected utility using the principle of insufficient reason? Show your calculations of the expected utilities of the options, or mathematically equivalent calculations that you may choose to use instead. (Either way, you do not have to reduce any fractions.)

   answer:

   $$\text{EU}(A_1) = \left(\frac{1}{3}\right)(8) + \left(\frac{1}{3}\right)(5) + \left(\frac{1}{3}\right)(4) = \frac{8}{3} + \frac{5}{3} + \frac{4}{3} = \frac{17}{3}$$

   $$\text{EU}(A_2) = \left(\frac{1}{3}\right)(6) + \left(\frac{1}{3}\right)(1) + \left(\frac{1}{3}\right)(7) = \frac{6}{3} + \frac{1}{3} + \frac{7}{3} = \frac{14}{3}$$

   $$\text{EU}(A_3) = \left(\frac{1}{3}\right)(2) + \left(\frac{1}{3}\right)(3) + \left(\frac{1}{3}\right)(9) = \frac{2}{3} + \frac{3}{3} + \frac{9}{3} = \frac{14}{3}$$

A_1 has the highest expected utility, so it is the option that the rule of maximizing expected utility using the principle of insufficient reason would recommend.

9. Suppose a potential arsonist has preferences that can be represented with the following utility function:

item	utility
committing arson	1.5
not committing arson, spending 0 years in prison	0
spending 0.25 years in prison	–0.38
spending 0.5 years in prison	–0.62
spending 0.75 years in prison	–0.82
spending 1 year in prison	–1.0
spending 2 years in prison	–1.6
spending 3 years in prison	–2.2
spending 4 years in prison	–2.6
spending 5 years in prison	–3.1
spending 10 years in prison	–5.0
spending 20 years in prison	–8.1
spending 40 years in prison	–13

Suppose the punishment for arson is spending 10 years in prison and the probability of the implementation of that punishment, in any case of arson, is p. Also, assume that if the potential arsonist does not commit arson, he will not spend any time in prison. What must the potential arsonist believe about p in order for him to judge that the expected utility of committing arson is lower than the expected utility of not committing arson? Write your answer as an equation or inequality of one of the following three forms:

$p =$ __ or $p >$ __ or $p <$ __

Of course, on the right side of your answer, instead of a blank ('__'), you will have a number or a numerical expression. That expression does not have to be simplified. For example, it could be some numbers (possibly integers, but not necessarily) that are added, subtracted, multiplied, and/or divided by each other. But the left side of your answer must be just 'p', and the middle must be just '=' or '>' or '<'.

answer:

$$EU(\text{arson}) < EU(\text{no arson})$$

$$(p)(1.5 - 5.0) + (1 - p)(1.5) < u(\text{no arson})$$

$$1.5p - 5.0p + 1.5 - 1.5p < 0$$

$$-5.0p + 1.5 < 0$$

$$-5.0p < -1.5$$

$$5.0p > 1.5$$

$$p > \frac{1.5}{5.0}$$

(or anything equivalent to $p > \frac{3}{10}$)

10. Suppose all of the following: (1) you are taking a test consisting of multiple-choice questions that have nine answer choices each, (2) answering a question correctly increases your raw score by some number of points, (3) answering a question incorrectly decreases your raw score by 2 points, and (4) not answering a question does not increase or decrease your raw score. How much does answering a question correctly need to increase your raw score in order for guessing randomly among the nine answer choices to have a higher expected value (in terms of your raw score on the test) than not answering?

    answer:

    Let *x* be the number of points by which your raw score is increased if you answer a question correctly. Then we solve as follows:

$$EV(\text{guess}) > EV(\text{blank})$$

$$\left(\frac{1}{9}\right)(x) + \left(\frac{8}{9}\right)(-2) > 0$$

$$\frac{x}{9} - \frac{16}{9} > 0$$

$$\frac{x - 16}{9} > 0$$

$$x - 16 > 0$$

$$x > 16$$

    So, answering a question correctly needs to increase your raw score by more than 16 points in order for guessing randomly among the nine answer choices to have a higher expected value (in terms of your raw score on the test) than not answering.

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.